however a nonequilibrium phenomenon. The magnetic phase diagram of a Type II superconductor is shown in Fig. 8.14. These predictions spurred the search for materials with high $H_{c_2}$. Many such materials were synthesised, and after years of development effort, made into wires and tapes for stable high field superconducting magnets. It turns out that the new copper oxide superconductors are extreme Type II materials with an $H_{c_2}$ of order 150 T. A material can change from Type I to Type II on the substitution of some impurities. For example, lead is a Type I superconductor with $H_c = 4.8 \times 10^4$ A/m at 4 K. When 2 wt% indium is added to it, it becomes as Type II superconductor with $H_{c_1} = 3.18 \times 10^4$ A/m and $H_{c_2} = 7.96 \times 10^4$ A/m. On adding 20 wt% of indium in lead, $H_{c_1} = 5.6 \times 10^3$ A/m and $H_{c_2} = 2.9 \times 10^5$ A/m.



## XIV. LONDON EQUATIONS: ELECTRODYNAMICS

The London theory is based on rather the old ideas of the *two fluid model*, according to which a superconductor can be thought to be composed of both normal and superfluid electrons. The Maxwell's electromagnetic equations are inadequate to explain the *electrodynamics* of superconductors, namely, the zero resistance state coupled with perfect diamagnetism. The London brothers in 1935 derived two field equations though successful in explaining the two important experimental facts about superconductors, do not give any insight into the underlying electronic process in superconductors anything more than what Ohm's law does for normal conductors. In this respects London's theory is purely macroscopic.

According to London's theory, it is assumed that there are two types of conduction electrons in a superconductor, namely, the *superelectrons* and the *normalelectrons*. At 0 K a superconductor contains only superconducting electrons, but as temperature increases the ratio of the normal electrons to superconducting electrons increases, until at the transition temperature all the electrons are normal. At any temperature the sum of the superconducting electrons and the normal electrons is equal to the conduction electron density in the material in the normal state. The superconducting electrons are not subjected to any lattice scattering and therefore are merely accelerated in electric field.

Let $n_n \mathbf{v}_n$ and $n_s \mathbf{v}_s$ be respectively the density, and velocity of the normal and superfluid electrons. If $n$ is the number of electrons $m^3$ then on the average

$$n = n_n + n_s$$

The current density is given by

$$\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s = en_n \mathbf{v}_n + en_s \mathbf{v}_s \qquad (8.12)$$

In a superconducting metal the superconducting electrons encounter no resistance to their motion, so, if a constant electric field $\mathbf{E}$ is maintained in the material, the electrons accelerate steadily under the action of this field.

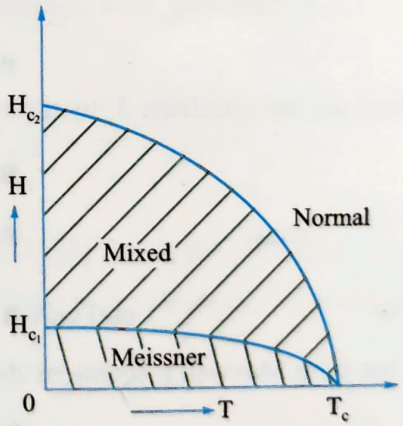


Fig. 8.15 Magnetic phase diagram of a type II superconductor

The equation of motion for the superconducting electron is

$$m(d\mathbf{v}_s/dt) = e\mathbf{E}$$

*i.e.*,

$$m\dot{\mathbf{v}}_s = e\mathbf{E}$$

The *supercurrent density* is

$$\mathbf{J}_s = en_s\mathbf{v}_s \text{ and } (d\mathbf{J}_s/dt) = (en_s)\left[\frac{d\mathbf{v}_s}{dt}\right]$$

$$\dot{\mathbf{J}}_s = n_s e\dot{\mathbf{v}}_s = (n_s e^2\mathbf{E})/m \tag{8.13}$$

*i.e.*,

This is the first London equation which describes the absence of resistance. Since no electric field is necessary unless the current changes, it becomes possible to have steady currents in the absence of electric field. This is the superconducting phenomenon. The corresponding equation for the normal current is

$$\mathbf{J}_n = \sigma_n\mathbf{E} = \frac{n_n e^2\tau\mathbf{E}}{m} \tag{8.14}$$

This equation says that an electric field is necessary for establishing a steady current. If $\mathbf{E} = 0$, $\mathbf{J}_n$ becomes zero unlike the superconducting state.

A magnetic field is related to an electric field and current by Maxwell's equation as

$$\dot{\mathbf{B}} = -\text{ curl } \mathbf{E} = -\mu_0\dot{\mathbf{H}} \tag{8.15}$$

Again from Maxwell's equations,

$$\text{curl } \mathbf{H} = \mathbf{J}_s + \dot{\mathbf{D}} \tag{8.16}$$

Now,

$$\text{curl } \mathbf{B} = \mu_0(\mathbf{J}_s + \dot{\mathbf{D}}) \tag{8.17}$$

where $\mathbf{J}_s$ is the current density within the metal. Furthermore, unless the fields are varying very rapidly with time, the displacement current $\dot{\mathbf{D}}$ is negligible in comparison with $\mathbf{J}_s$. Hence inside a superconductor we can write Maxwell's equations in the form

$$\dot{\mathbf{B}} = -\text{ curl } \mathbf{E} \tag{8.18}$$

and

$$\text{curl } \mathbf{B} = \mu_0\mathbf{J}_s \tag{8.19}$$

Substituting from equation (8.13) into equation (8.18), one gets

$$\dot{\mathbf{B}} = -(m/n_s e^2)\text{ curl } \dot{\mathbf{J}}_s \tag{8.20}$$

and we can eliminate $\mathbf{J}_s$ by means of equation (8.19), we get

$$\dot{\mathbf{B}} = -(m/\mu_0 n_s e^2)\text{ curl curl } \dot{\mathbf{B}} \tag{8.21}$$

Thus,

$$\dot{\mathbf{B}} = -\alpha\text{ curl curl } \dot{\mathbf{B}} \text{ with } \alpha = \frac{m}{\mu_0 n_s e^2} \tag{8.22}$$

or

$$\text{curl curl } \dot{\mathbf{B}} = \text{grad div } \dot{\mathbf{B}} - \nabla^2\dot{\mathbf{B}} \tag{8.23}$$

but from Maxwell's equations $\text{div } \dot{\mathbf{B}} = 0$, so equation (8.22) becomes

$$\dot{\mathbf{B}} = \alpha\nabla^2\dot{\mathbf{B}}$$

or

$$\nabla^2\dot{\mathbf{B}} = \frac{\dot{\mathbf{B}}}{\alpha} \tag{8.24}$$

This is a differential equation which **B** must satisfy. In order to study the implication of this, consider the plane boundary of a superconductor with a uniform magnetic field applied parallel to the boundary (Fig. 8.15*a*). Suppose that the flux density outside the metal is $\mathbf{B}_a$ and let the direction normal to the boundary be called the x-direction. Because the applied field is uniform, **B** will have the same direction everywhere, so may regard equation (8.24) as a scalar one; also there will be no gradients of the field parallel to the boundary, so in this case equation (8.24) reduces to

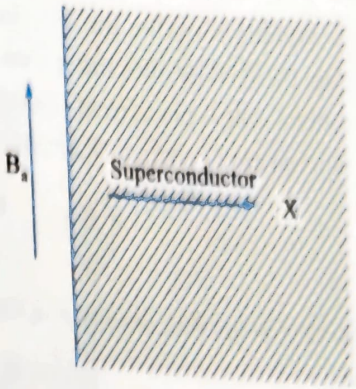


Fig. 8.15 (*a*) Magnetic field applied parallel to the boundary of the superconductor

$$\frac{\partial^2 \dot{\mathbf{B}}}{\partial x^2} = \frac{\dot{\mathbf{B}}}{\alpha}$$

The solution of this equation is of the form

$$\dot{\mathbf{B}}(x) = \dot{\mathbf{B}}_a \exp(-x/\sqrt{\alpha}) \qquad (8.24a)$$

where $\dot{\mathbf{B}}(x)$ is the flux density at a distance $x$ inside the metal and $\dot{\mathbf{B}}_a$ is the value of $\dot{\mathbf{B}}$ outside the metal. This means that $\dot{\mathbf{B}}$ dies away exponentially as we penetrate into the superconductor. In other words, changes in flux density do not penetrate far below the surface, so at a sufficient distance inside the metal the flux density has a constant value which does not change with time, irrespective of what is happening to the applied field.

We have deduced the above behaviour by applying the usual laws of electrodynamics to a conductor with zero resistance; however, though equation (8.24*a*) completely describes the magnetic properties of perfect conductor, it does not adequately, describe the behaviour of a superconductor. The Meissner effect shows that inside a superconductor the flux density is not only constant but the value of this constant is always zero; so not only $\dot{\mathbf{B}}$ but **B** itself must die away rapidly below the surface. F. London and H. London suggested that the magnetic behaviour of a superconducting metal might be correctly described if equation (8.24) applied not only to $\dot{\mathbf{B}}$ but **B** itself,

$$\nabla^2 \mathbf{B} = \frac{\mathbf{B}}{\alpha} \qquad (8.25)$$

The solution is of the form [Refer equation (8.24*a*)]

$$\mathbf{B}(x) = \mathbf{B}_a \exp(-x/\sqrt{\alpha}) \qquad (8.26)$$

Examination of the argument by which we derived [equation (8.24)] shows that we could have derived equation (8.25) if everywhere we had replaced $\dot{\mathbf{B}}$ by **B**. If we retrace the argument, we arrive at equation (8.20) which would now have the more restrictive form

$$\mathbf{B} = -\frac{m}{n_s e^2} \text{curl } \mathbf{J}_s \qquad (8.27)$$

This equation and equation (8.13), namely

$$\dot{\mathbf{J}}_s = \frac{n_s e^2}{m} \mathbf{E} \qquad (8.28)$$

which together describe the electrodynamics of the supercurrent are known as *London equations*. Equation (8.28) describes the resistanceless property of a superconductor, there being no electric field in the metal unless the current is changing; and equation (8.27) describes the diamagnetism. For a one-dimensional case equation (8.25) takes the form

$$\frac{\partial^2 \mathbf{B}(x)}{\partial x^2} = \frac{\mathbf{B}(x)}{\alpha}$$

where, $\mathbf{B}(x)$ is the flux density at a distance $x$ inside the metal. This equation has the solution

$$\mathbf{B}(x) = \mathbf{B}_a \exp(-x/\sqrt{\alpha}) \tag{8.29}$$

where, $\mathbf{B}_a$ is the flux density of the applied field at the surface. Equation (8.29) indicates that the flux density decays exponentially in a superconductor, falling $(1/e)$ of its value at the surface at a distance $x = \sqrt{\alpha}$. This distance is called the *London penetration depth*, $\lambda$.

Now $\alpha = m/\mu_0 n_s e^2$, so the London penetration depth

$$\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}} \tag{8.30}$$

$\lambda$ will be of the order of 1 nm.

We can now write the London equations (8.27) and (8.28) as

$$\text{curl } \mathbf{J}_s = -\frac{\mathbf{B}}{\mu_0 \lambda^2} \tag{8.31}$$

$$\dot{\mathbf{J}}_s = \frac{\mathbf{E}}{\mu_0 \lambda^2} \tag{8.32}$$

London equations do not replace Maxwell's equations, but are additional conditions obeyed by the supercurrents. The penetration depth is found to depend strongly on temperature and to become much larger as $T$ approaches $T_c$. At low temperatures, it is nearly independent of temperature and has a value $\lambda_0$ characteristic of the particular metal. Above about 0.8 of the transition temperature, however, the penetration depth increases rapidly and approaches infinity as the temperature approaches the transition temperature (see Fig. 8.13). The observation can be fitted well by a simple expression of the form.

This equation implies that

$$\frac{\lambda(T)}{\lambda(0)} = \left[1 - \left\{\frac{T}{T_c}\right\}^4\right]^{-\frac{1}{2}} \tag{8.33}$$

$$n_s = n_0\left[1 - \left\{\frac{T}{T_c}\right\}^4\right] \tag{8.34}$$